SUMMATIVE ASSESSMENT – II, 2015, MATHEMATICS, Class – IX

SOLVED SAMPLE QUESTION PAPER

JST201503

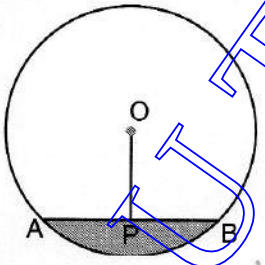
Time allowed: 3 hours **Maximum Marks: 90**

General Instructions :

1. **All** questions are compulsory.
2. The question paper consists of **31 questions** divided into four **sections** A, B, C and D. **Section 'A'** comprises of **4 questions** of **1 mark** each, **Section 'B'** comprises of **6 questions** of **2 marks** each, **Section 'C'** comprises of **10 questions** of **3 marks** each and **Section 'D'** comprises of **11 questions** of **4 marks** each.
3. There is no overall choice.
4. Use of calculator is not permitted.

SECTION — A

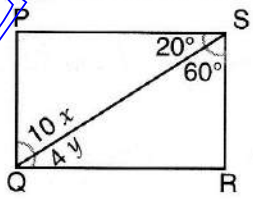
1. At what point the graph of the linear equation $4x - 3y = 12$ cuts y-axis ?
2. In figure, O is the centre of the circle and $PA = PB$. Find $\angle OPA$.



3. Find two points that lie on the graph of the linear equation $2x + y + 5 = 0$.
4. Find the range of the data :
   25, 18, 20, 22, 16, 6, 17, 12, 30, 32, 10, 8, 11, 20

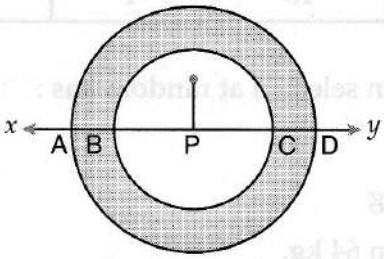
SECTION — B

5. In the given figure $PQRS$ is a parallelogram. Find the value of x and y.



6. What is the volume of right circular cylinder, whose base area is 606 cm^2 and height is 2 m ?
7. If the length of a diagonal of a cube is $16\sqrt{3}$. Find the volume of the cube.
8. Find the mode of the observation : 3, 5, 7, 4, 7, 8, 3, 6, 7, 4, 7, 3. If 5 is added to each observation, what will be the new mode ?
9. Find the value of $3x + 1$, if median of 2, 3, x, $x + 2$, 11, 17 is 9. (The observations are arranged in ascending order of magnitude.)

10. If a line intersects two concentric circles with common centre O, at A, B, C and D. Prove that $AB = CD$.



11. Find whether the given ordered pair is a solution of the given linear equations :

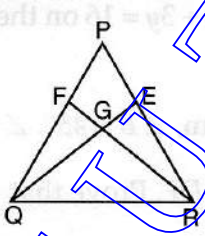
(a) $2x - 4y = 32$; $(8, -4)$ (b) $4x - 2y = 10$; $(3, -1)$

(c) $2x = 5$; $(0, 5)$

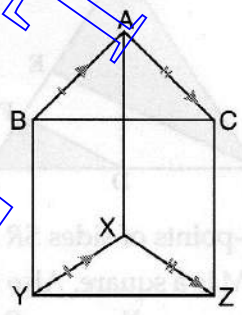
12. Draw a triangle whose sides are represented by $x = 0$, $y = 0$ and $x + y = 3$, in the cartesian system. Also find the co-ordinate of its vertices.

13. Construct an equilateral triangle of side 6.6 cm. Label its vertices as A, B and C. From point C, draw a median CF.

14. The medians QE and RF of ΔPQR intersect at G. Prove that ar (GQR) = ar $(PFGE)$.



15. In the given figure $AB = XY$, $AC = XZ$, $AB \| XY$ and $AC \| XZ$. Show that $\Delta ABC \cong \Delta XYZ$



16. The surface area of a sphere of radius 5 cm is five times the curved surface area of a cone of radius 4 cm. Find the height of the cone.

17. The total cost of making a solid spherical ball is ₹ 33,957 at the rate of ₹ 7 per cubic meter. Find the radius of this ball.

18. For the data 3, 21, 25, 17, $(x + 3)$, 19, $(x - 4)$ if mean is 18, find the value of x and hence, find the mode of the data.

19. Three coins are tossed simultaneously 1000 times and the following observations are made. Three heads = 216 times, two heads = 384 times, 1 head = 270 times, no head = 130 times. If coins are tossed once again, find the probability of :

(a) non occurrence of exactly 2 heads (b) 3 heads (c) no heads .

20. The weights of 60 persons in a group are given below :

Weight (in kg)	60	61	62	63	64	65
No. of persons	5	18	4	16	5	12

Find the probability that a person selected at random has :

(i) weight less than 65 kg

(ii) weight between 61 and 64 kg

(iii) weight equal to or more than 64 kg.

21. The monthly hostel charges for a student comprises of ₹ 1000 p.m. as fixed boarding charges and remaining charges at the rate of ₹ 50 per day (for the no. of days for which the food has been availed by a student)

(a) Form a linear equation in two variables to represent the above situation.

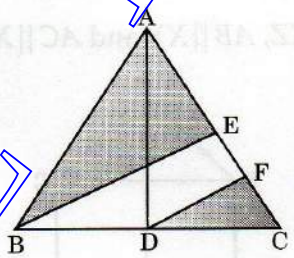
(b) Find two solutions possible for the equation formed.

(c) What are the monthly charges to be paid by a student who availed meals for 21 days in given month ?

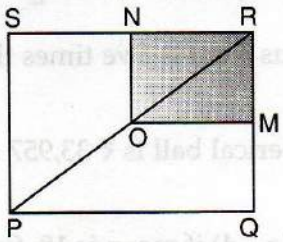
22. Draw the graphs of line $x + y = 6$ and $2x + 3y = 16$ on the same graph. Also find the co-ordinates of the point where two lines intersect.

23. Construct a ΔABC with perimeter 16 cm, $\angle B = 45°, \angle C = 30°$.

24. In figure, AD and BE are medians $BE \| DF$. Prove that $CF = \frac{1}{4}AC$.



25. PQRS is a square. N and M are the mid-points of sides SR and QR respectively. O is a point on diagonal PR such that $OP = OR$. Show that $ONRM$ is a square. Also find the ratio of ar (ΔORM) and ar ($PQRS$).



26. Prove that the opposite angles of an isosceles trapezium are supplementry.

27. Prove that equal chord of a circle subtend equal angles at the centre.

28. Rain water which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if the rain fall is 1 cm ? Give your answer to the nearest whole number. (use $\pi = 3.14$)

29. A small indoor green house is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high. What is the area of the glass ? How much tape of width 10 cm is required for all the 12 edges ?

30. Draw a histogram and frequency polygon for the following data :

C.I.	0 – 50	50 –100	100 – 150	150 – 200	200 – 250	250 – 300
Frequency	12	18	27	20	17	6

(i) Which mathematical concept is used in the above problem ?

(ii) What is its value ?

31. Naveen was having a plot in the shape of a quadrilateral. He decided to donate some portion of it to construct a home for orphan girls. Further he decided to buy a land in lieu of portion, his donated this plot so as to form a triangle.

(i) Explain how this proposal will be implemented.

(ii) Which mathematical concept is used in the above problem ?

(iii) Which values are depicted by Naveen ?

Solution:

1. Equation $4x - 3y = 12$

Put $x = 0$ in equation ½

$$4 \times 0 - 3y = 12$$
$$0 - 3y = 12$$
$$y = \frac{12}{-3} = -4$$

Hence, graph of linear equation cut the y-axis at $(0, -4)$. ½

2. In a circle the line joining the mid-point of a chord of centre is perpendicular to the chord. **1**

$\therefore \quad \angle OPA = 90°$

3. Equation $2x + y + 5 = 0$

Put $x = 1$ in equation

$$2 \times 1 + y + 5 = 0$$
$$y + 7 = 0$$
$$y = -7$$

$\therefore$ Point $= (1, -7)$. ½

Again, put $x = -1$ in equation

$$2 \times (-1) + y + 5 = 0$$
$$-2 + y + 5 = 0$$
$$y + 3 = 0$$
$$y = -3$$

$\therefore$ Point $= (-1, -3)$. ½

4. Range = Highest value – lowest value
= 32 – 6 = 26. **1**

5. Since PQRS is a parallelogram

$PQ \parallel SR$, $PS \parallel QR$ and QS is a transversal

$4y = 20° \quad y = 5° \quad 10x = 60° \quad x = 6°$

6. Given, Area $= 606\ cm^2$.

$h = 2\ m = 200\ cm.$ ½

$\therefore$ Volume $= \pi r^2 h$

$= (Area)h\ cu.cm$ ½

$= 606 \times 200$

$= 121200\ cm^3.$ **1**

(CBSE Marking Scheme, 2014)

7. Let the length of a cube is a then length of a diagonal of a cube $= \sqrt{3}a = 16\sqrt{3}$

$a = 16\ cm$ **1**

Volume $= a^3 = 16^3$

$= 4096$

8. **Observation :** 3, 5, 7, 4, 7, 8, 3, 6, 7, 4, 7, 3

Mode of observation $= 7$ **1**

After adding,

New observation : 8, 10, 12, 9, 12, 13, 8, 11, 12, 9, 12, 8

New mode $= 12$ **1**

9. **Observation :** 2, 3, x, $x + 2$, 11, 17

$N = 6$ (even)

$$\text{Median} = \frac{\left(\frac{N}{2}\right)^{th}\text{term} + \left(\frac{N+1}{2}\right)^{th}\text{term}}{2}$$ ½

$$9 = \frac{\left(\frac{6}{2}\right)^{th}\text{term} + \left(\frac{6}{2}+1\right)^{th}\text{term}}{2}$$

$18 = 3^{th}\text{ term} + 4^{th}\text{ term}$ ½

$18 = x + x + 2$ ½

$2x = 16$

$x = 8$

$\therefore$ Value of $3x + 1 = 3 \times 8 + 1 = 25$ ½

10. Draw OP perpendicular to xy from the centre to a chord bisects it.

$OP \perp$ to chord BC

$\Rightarrow$ $BP = PC$ ...(i) **1**

Similarly, $AP = PD$ ... (ii)

From (i) and (ii), we get $PD - BP = PD - PC$ ½

$\therefore$ $AB = CD.$ ½

11. **(a)** $2x - 4y = 32$

Put (8, – 4) in equation

L.H.S. $= 2 \times 8 - 4 \times (-4) = 16 + 16$

$= 32 =$ R.H.S.

Hence, (8, – 4) is a solution of given linear equation. **1**

(b) $4x - 2y = 10$

Put (3, – 1) in equation

L.H.S. $= 4 \times 3 - 2 \times (-1) = 12 + 2$

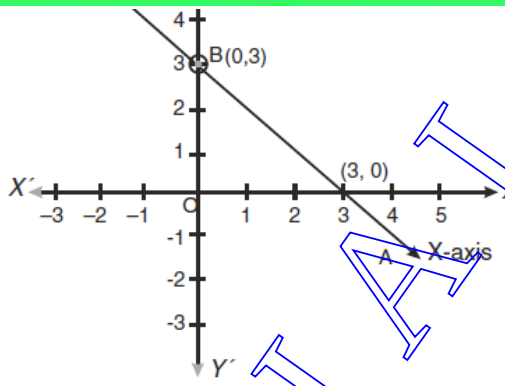
$= 14 \neq$ R.H.S.

Hence, (3, – 1) is not a solution of given linear equation. **1**

(c) $2x = 5$ Put $x = 0$ in equation L.H.S. $= 2 \times 0 = 0 \neq$ R.H.S.

Hence, (0, 5) is not a solution of given equation.

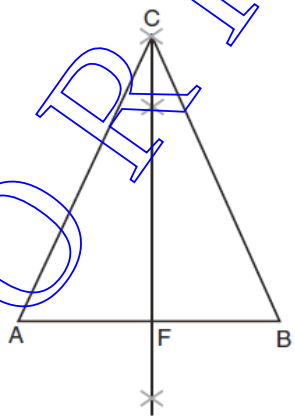
12. xx' line represented the line $y = 0$
yy' line represented the line $x = 0$



Put $x = 0$ in equqation $x + y = 3$
$0 + y = 3$
$y = 3$
Again, put $y = 0$ in equation $x + y = 3$
$x + 0 = 3$
$x = 3$ **1**

ABC is the required triangle.
Co-ordinates of triangle ABC are (3, 0), (0, 3) and (0, 0). **1**

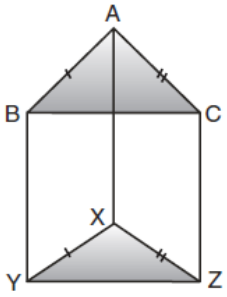
13. Steps of construction : **1½**



(i) Draw any line segment AB = 6·6 cm.
(ii) With A as centre and radius 6·6 cm draw an arc.
(iii) With B as centre and radius 6·6 cm draw an arc to cut the previous arc at C.
(iv) Join AC and AB, then ABC is the required triangle.
(v) Again, draw perpendicular bisector of AB which cut AB at F.
(vi) Join C to F which is median. **1½**

14. $\text{ar}(\Delta QER) = \text{ar}(\Delta QEP)$ [QE is median]
$\text{ar}(\Delta QR) + \text{ar}(\Delta ER) = \text{ar}(PFGE) + \text{ar}(\Delta GFQ)$ ...(i) **1**
Also $\text{ar}(\Delta PER) = \text{ar}(\Delta QFR)$
$\text{ar}(\Delta FGE) + \text{ar}(\Delta EGR) = \text{ar}(\Delta GQR) + \text{ar}(\Delta GFQ)$ ...(ii) **1**
From (i) – (ii) $\text{ar}(\Delta GQR) - \text{ar}(PFGE) = \text{ar}(PFGE) - \text{ar}(\Delta GQR)$
$\text{ar}(GQR) = \text{ar}(PFGE)$ **1**

15. In quadrilateral $ABYX$
$AB = XY$ and $AB \parallel XY$ **1**



$\because$ one pair of opposite side of a quadrilateral are equal and parallel.
$\therefore$ ABYX is a parallelogram ...(i)
Similarly, ACZX is a parallelogram ...(ii)
From (i) and (ii), $BY \parallel CZ$ and $BY = CZ$
$\therefore$ $BCZY$ is also a parallelogram. **1**

$BC = YZ$ and $BC \parallel YZ$

In ΔABC and ΔXYZ

$AB = XY$ (given)
$AC = XZ$ (given)
$BC = YZ$ (proved above)
$\therefore$ $\Delta ABC \cong \Delta XYZ$ **1**

16. Radius of sphere (r) = 5 cm
Radius of cone (R) = 4 cm
According to question Surface area of sphere
$= 5 \times$ curved surface area of cone
$4\pi r^2 = 5 \times \pi Rl$ **1**

$$l = \frac{4r^2}{5R} = \frac{4\times5\times5}{4\times5}$$

$$l = 5 \text{ cm} \quad 1$$

Height of cone, $h = \sqrt{l^2 - R^2} = \sqrt{5^2 - 4^2}$

$= \sqrt{25-16} = \sqrt{9} = 3$ cm **1**

17. Let the radius of this ball $= r$ m

Volume of spherical ball $= \frac{4}{3}\pi r^3$

The total cost of making a solid spherical ball $= 7 \times \frac{4}{3}\pi r^3$

$$33957 = 7 \times \frac{4}{3} \times \frac{22}{7} \times r^3 \quad 1$$

$$r^3 = \frac{33957\times3}{4\times22} = \frac{343\times9\times3}{8} \quad 1$$

$$r^3 = \left(\frac{7\times3}{2}\right)^3$$

$$r = \frac{21}{2} = 10{\cdot}5 \text{ m} \quad 1$$

18. **Data :** 3, 21, 25, 17, $(x + 3)$, 19, $(x - 4)$

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observation}} \quad 1$$

$$18 = \frac{3+21+25+17+x+3+19+x-4}{7}$$

$$126 = 88 - 4 + 2x$$

$$2x = 126 - 84 = 42$$

$$x = \frac{42}{2} = 21 \quad 1$$

Data : 3, 21, 25, 17, (21 + 3), 19, (21 – 4) = 3, 21, 25, 17, 24, 19, 17

Mode = 17 **1**

19. **(a)** Probability (non occurrence of exactly 2 heads)

$$= \frac{216+270+130}{1000} = \frac{616}{100} = 0{\cdot}616 \quad 1$$

(b) Probability (3 heads) $= \frac{216}{1000} = 0{\cdot}216$ **1**

(c) Probability (no head) $= \frac{130}{1000} = 0{\cdot}13$ **1**

20. **(i)** P (weight less than 65 kg) $= \frac{5+18+4+16+5}{60} = \frac{48}{60} = \frac{4}{5}$

(ii) P (weight between 61 and 64) $= \frac{4+16}{60} = \frac{20}{50} = \frac{1}{3}$

(iii) P (weight equal to or more than 64) $= \frac{5+12}{60} = \frac{17}{60}$ **(1 + 1 + 1)**

21. **(a)** Fixed charges = ₹ 1,000

Let the no. of days for which the food has been availed = y

Let total charges = x

Then according to question $x = 1000 + 50y$ ...(i) **1**

(b) Put $y = 4$ (days) in equation (i)

$x = 1000 + 50 \times 4$

$x = 1000 + 200$

$x = 1200$ **1**

Again, put $y = 6$ (days) in equation (i)

$x = 1000 + 6 \times 50$

$= 1000 + 300$

$= 1300$ **1**

Hence, (1200, 4) and (1300, 6) are the two solutions.

(c) Again, put $y = 21$ (days) $x = 1000 + 50 \times 21$

$= 1000 + 1050$

$x =$ ₹ 2,050 charges for 21 days. **1**

22. $x + y = 6$

$y = 6 - x$ ...(i)

Put $x = 1$ in equation (i) $y = 6 - 1 = 5$

Put $x = 2$ in equation (i) $y = 6 - 2 = 4$

Put $x = 3$ in equation (i) $y = 6 - 3 = 3$

x	1	2	3
y	5	4	3

1½

$2x + 3y = 16$

$y = \frac{16 - 2x}{3}$ ...(ii)

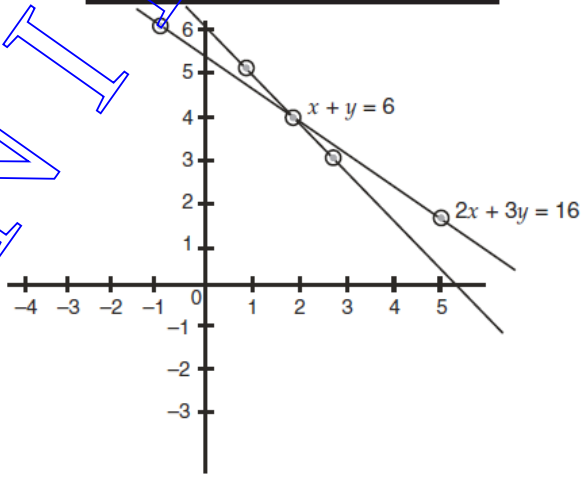
Put $x = 2$ in equation (ii) $y = \frac{16 - 2 \times 2}{3} = \frac{12}{3} = 4$

Put $x = 5$ in equation (ii) $y = \frac{16 - 2 \times 5}{3} = \frac{6}{3} = 2$

Put $x = -1$ in equation (i) $y = \frac{16 - 2 \times (-1)}{3} = \frac{16 + 2}{3} = 6$

x	2	5	–1
y	4	2	6

½



1

Intersecting point of two line = (2, 4).

23. **Steps of Construction,** **2½**

(i) Draw a line $PQ = 16$ cm.

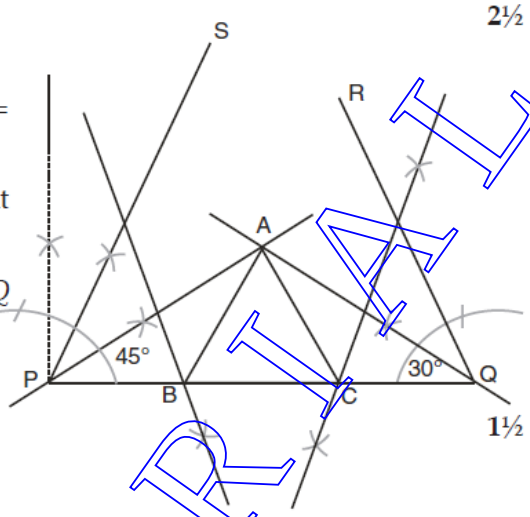
(ii) At P, construct $\angle SPQ = 45°$ and at Q construct $\angle RQP = 30°$.

(iii) Draw the bisectors of $\angle SPQ$ and $\angle RQP$, intersecting at A.

(iv) Draw the right bisector of AP and AQ intersecting PQ at B and C respectively.

(v) Join A to B and A to C.

ABC is the required triangle



1½

24. In ΔBEC, $BE \parallel DF$ and D is the mid-point of BC.

$\therefore$ F is the mid-point of CE

$$\therefore \quad CF = \frac{1}{2}\text{CE} \qquad \text{...(i) 1}$$

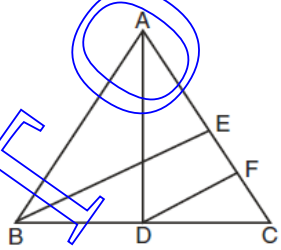
As, BE is the median, E is the mid-point of AC

$$CE = \frac{1}{2}AC \qquad \text{...(ii) 1}$$

From (i) and (ii), we get **1**

$$CF = \frac{1}{2}\left(\frac{1}{2}AC\right)$$

$$CF = \frac{1}{4}AC \qquad \textbf{1}$$



25. Since $OP = OR$, O is the mid-point of PR.

In SRP, O and N are the mid-points of PR and SR respectively.

By mid-point theorem,

$$ON = \frac{1}{2}SP \text{ and } ON \parallel SP \qquad \text{...(i) 1}$$

Similarly, $OM \parallel PQ$ ...(ii)

Using (i) and (ii) we get,

ONRM is a $\|^{gm}$

Now, $ON = \frac{1}{2}SP$

$= \frac{1}{2}SR \quad (\because SP = SR)$

$= NR$

In $\|^{gm}$ $ONRM$, pair of adjacent sides ON and NR are equal and

$\angle S = \angle N = 90°$ (Corresponding angles)

$\therefore$ $ONRM$ is a square.

Since OR is a diagonal of square.

$$\text{ar}(\Delta ORM) = \text{ar}(\Delta ONRM) \qquad \text{...(iii)}$$

$$\text{ar}(ONRM) = NR \times RM$$

$$= \frac{1}{2}SR \times \frac{1}{2}RQ = \frac{1}{4}(SR)^2$$

$$= \frac{1}{4}\text{ ar}(PQRS) \qquad \text{...(iv) 1}$$

Using (iii) and (iv) we get,

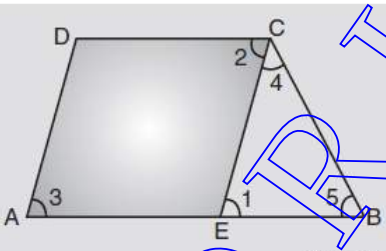
$$\frac{\text{ar}(\Delta ORM)}{\text{ar}(PQRS)} = \frac{\frac{1}{2}\text{ar}(ONRM)}{4\,\text{ar}(ONRM)}$$

$$= \frac{1}{8}$$

$\therefore$ The ratio of ar(ΔORM) : ar($PQRS$) $= 1 : 8$ **1**

26. In trapezium $ABCD$,

$AB \parallel DC$ and $AD = BC$



Through C, draw $CE \parallel DA$

$DC \parallel AE$ and CE is transverse

$\therefore$ $\angle 1 = \angle 2$ (alternate angles)

Also, $\angle 3 = \angle 1$ (Corresponding angles) **2**

$\therefore$ $\angle 2 + \angle 3 = \angle 1$

$\angle 2 + \angle 3 = 2\angle 1$

$\therefore$ $\angle A + \angle C = \angle 3 + \angle 2 + \angle 4$

$= 2\angle 1 + \angle 4$ ...(i)

Also, $\angle 1 = \angle 5$ ($\because EC = BC$) **1**

$\angle A + \angle C = \angle 1 + \angle 4 + \angle 5 = 180°$

Similarly, we can show that

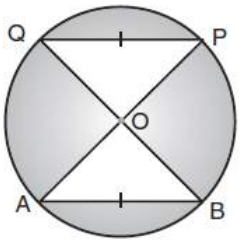
$\angle B + \angle D = 180°$.

Hence, the opposite angles of an isosceles trapezium are supplementary. **1**

(CBSE Marking Scheme, 2014)

27. Chord $AB = PQ$ (given)

In ΔAOB and ΔCOD



$OA = OP$ (radii of circle) **1**

$OB = OQ$ (radii of circle) **1**

$AB = PQ$ (given)

$\Delta AOB \cong \Delta QOP$ (by S.S.S.) **1**

$\therefore$ $\angle APB = \angle POQ$ (by C.P.C.T.) **1**

28. Volume of water which is transferred into a cylindrical vessel = lbh

$= 6\text{ m} \times 4\text{ m} \times 1\text{ cm}$

$= 600 \times 400 \times 1$ **1½**

$= 240000\text{ cm}^3$

Let the height of water in cylindrical vessel = h cm

Then volume of water = volume of cylindrical vessel **1**

$$240000 = \pi r^2 h = \frac{22}{7} \times 20 \times 20 \times h$$

$$h = \frac{240000 \times 7}{22 \times 20 \times 20} = 190{\cdot}9\text{ cm}$$ **1½**

29. Here $l = 30$ cm, $b = 25$ cm, $h = 25$ cm

Area of glass = Total surface area

$= 2(lb + bh + hl)$ **1**

$= 2[30 \times 25 + 25 \times 25 + 25 \times 30]$

$= 2[750 + 625 + 750]$

$= 2 \times 2125$

$= 4250 \text{ cm}^2$. **1**

Top needed for all the 12 edges = The sum of all the edges **1**

$= 4(l + b + h) = 4(30 + 25 + 25)$

$= 4 \times 80$

$= 320$ cm **1**

30. **(i)** For histogram,

y-axis = one square = 3

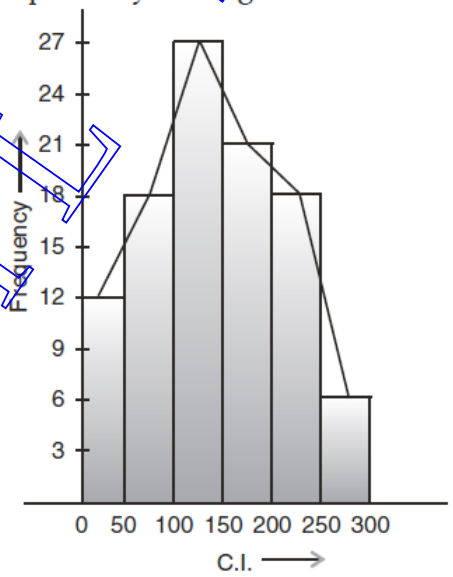
x-axis = one square = 50 **1**

(ii) For frequency polygon, first we obtain the class marks

C.I.	Class Marks	Frequency
0 — 50	25	12
50 — 100	75	18
100 — 150	125	27
150 — 200	175	20
200 — 250	225	17
250 — 300	275	6

1

To obtain the frequency polygon we plot the points (25, 12), (75, 18), (125, 27), (175, 20), (225, 17) and (275, 6) and join these points by line segment. **2**

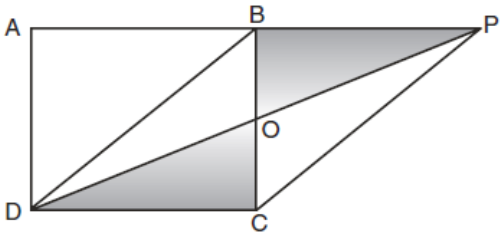


(i) Statistics. ½

(ii) Sincerity. ½

31. Let $ABCD$ be the plot and Naveen decided to donate some portion to construct as home for orphan girls from one corner say C of plot $ABCD$. Now, Naveen also purchases equal amount of land in lieu of land CDO, so that he may have triangular form of plot. BD is jointed. Draw a line through C parallel to DB to meet AB produced to P. **1**

Construction : Joint DP to intersect BC at O.



Proof : ΔBCD and ΔBPD are on the same base and between same parallels $CP \parallel DB$.

$\Rightarrow$ ar (ΔBCD) = ar (ΔBPD)

$\Rightarrow$ ar (ΔCOD) + ar (ΔDBO) = ar (ΔBOP) + ar $(\angle DBO)$

$\Rightarrow$ ar (ΔCOD) + ar (ΔBOP)

$\Rightarrow$ ar (quad. $ABCD$) = ar (quad. $ABOD$) + ar (ΔCOD) **1**

= ar (quad. $ABOD$) + ar (ΔBOP)

[$\therefore$ ar (ΔCOD) = ar (ΔBOP) proved above = ar (ΔAPD)

Hence, Naveen purchased the portion ΔBOP to meet his requirement. **1**

(ii) Area of parallelogram. ½

(iii) We should help the orphans. ½